

Experimental and Numerical Study for the Cross-flow around Four Cylinders in an In-line Square Configuration

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Abstract

The cross-flow around four cylinders in an in-line square configuration with the spacing ratio (L/D) 1.5, 2.5, 3.5 and 5.0 have been investigated experimentally using Laser Doppler Anemometer (LDA) and Digital Particle Image Velocimetry (DPIV). The experiments were carried out in a closed-loop wind tunnel with Reynolds number 1.128×10^4 to 1.982×10^4 . Mean velocity distributions are obtained by LDA. The full field instantaneous and averaged velocity and vorticity components are measured by DPIV. The present experimental study indicated that several distinct flow patterns exist. Distinct vortex shedding of the upstream cylinders was suppressed for $L/D < 3.5$ at $Re = 1.128 \times 10^4$. The flow patterns are affected by the spacing ratio and Re . In order to capture the details of the 3-D vortices structures and obtain all the instantaneous physical information, 3-D numerical simulations of the cross-flow around the four cylinders in an in-line square configuration with the spacing ratio 1.5 and 3.5, and $Re = 1.50 \times 10^4$ are carried out using large eddy simulation (LES). The numerical results are in good agreement with the experimental results. These results provided full field instantaneous information of the flow structures, velocity field and vorticity field of cross-flow around the four cylinders in an in-line square configuration.

Keywords: Four cylinders; Flow pattern; LDA; DPIV; LES

1. Introduction

The cross-flow around the cylinder arrays is very frequently present in the numerous different engineering applications, such as heat exchanges, offshore structures, etc. As a classical problem in fluid mechanics, cross-flow around the cylinder arrays exhibited many fundamental fluid dynamic phenomena due to the effect of free shear layer development, vortex shedding and wake interference, which depend on different conditions of the flow field such as the end conditions, blockage ratio of the flow

passage, the aspect ratio and spacing ratio of the bluff body, and Reynolds number, etc. The complexity of flow interference among the cylinder arrays has drawn considerable attention in the past.

The studies on one- or two-cylinder have been widely carried out experimentally and numerically for laminar and turbulent flows, such as Norberg (1998), Akilli et al. (2004), Sumner et al. (2000) and Park et al. (2003). The investigations have been summarized by Zdrav-kovich (1987) to different kinds of flow patterns for two cylinders in side-by-side arrangement and in tandem arrangement. Due to the complexity of the wake flow behind cylinder arrays, the flow around four-cylinder arrays configuration has not been

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studied as extensively as the simpler one and two cylinders configurations. However, a number of studies of the fluid dynamics for the four-cylinder arrays can still be found. For example, Sayers (1990) conducted experiments on four cylinders in a square configuration with two end plates in the spacing ratio L/D range of 1.5–5 and the aspect ratio $H/D=11.5$ using an inclined alcohol manometer and a hot-wire anemometer at $Re=3\times 10^4$, and the inclination angle varied from 0° to 180° at a 7.5° interval. They point out that at certain spacing ratios and inclination angles, the asymmetrical vortex shedding occurs, and in other certain conditions the vortex shedding may be absent. Farrant et al. (2000) captured numerically two-dimensional flow characteristics and interactive forces associated with flows around four equi-spaced cylinders at $Re=200$ using a cell boundary element method. It is interesting that for the in-line arrangement at $L/D=3.0$ case, computation result presented the shear layers from upstream cylinders roll up into mature vortices and impinge on the downstream cylinder surfaces instead of the shielding flow patterns behind the upstream cylinders from the experimental results suggested by Lam (1992). Lam et al. (2002; 2003; 2004; 1995; 1992) carried out extensively experimental investigations using laser induced fluorescent (LIF) visualization and Particle Image Velocimetry (PIV) for the cross-flow around four cylinders in different spacing ratios and different orientations at different Reynolds numbers.

However, the mean velocity and the instantaneous velocity field and the vorticity field for the cross-flow around four cylinders in an in-line square configuration at sub-critical Reynolds number have yet to be fully investigated. The main objectives of the present study are to investigate the effects of the spacing ratio and Re on the mean velocity distribution and the flow pattern around the four cylinders in an in-line square configuration using Laser Doppler Anemometer (LDA) and Digital Particle Image Velocimetry (DPIV). In order to capture the detailed 3-D vortices structures and obtain the full field instantaneous physical information, 3-D numerical simulations of the cross-flow around the four cylinders in an in-line square configuration with the spacing ratio 1.5 and 3.5 at $Re=1.50\times 10^4$ have been carried out using large eddy simulation (LES). A comparison of numerical and experimental results could lead to a further understanding of the effects of flow interference of the cross-flow around the four cylinders in an in-line

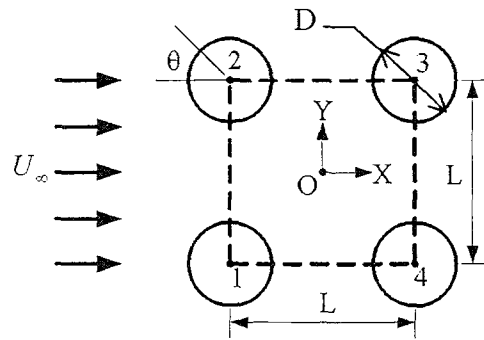


Fig. 1. Schematic diagram of the configuration for the four in-line cylinders.

square configuration.

2. Experimental arrangement

The experimental investigation on the cross-flow around the four cylinders in an in-line square configuration is conducted using a closed-circuit wind tunnel with a working square cross-section of $600\text{mm}\times 600\text{mm}$ and a length of 2000mm . The boundary layer on the wall was measured to be about 20mm thick, while the free-stream turbulent intensity was measured to be less than 2%. The four circular cylinders of diameter $D=25\text{mm}$ are placed horizontally with two moveable end-plates at the mid of the wind tunnel. The schematic diagram of the configuration for the four in-line cylinders is shown in Fig. 1. The origin of the coordinate system is located at the center point of the four cylinders arrangement, the (x, y, z) denoted the coordinates along the streamwise direction, the transverse direction and the spanwise direction of the cylinder, respectively. The spacing ratio (L/D) is set to 1.5, 2.5, 3.5 and 5.0 and the aspect ratio (H/D) is set to 20. H is the lengths of the cylinder along the spanwise direction. This leads to a blockage ratio (per cylinder) of 5%. The Reynolds number (Re) based on the diameter of the cylinder D and the free-stream velocity U_∞ varied from 1.128×10^4 to 1.982×10^4 . In order to obtain the mean velocity and the instantaneous full field velocity and vorticity information for the cross-flow around four cylinders in an in-line square configuration, a two-color fiber-optic LDA with a focal length of 310mm and a digital PIV were employed in the present study, respectively. The LDA measurement was carried out at difference spanwise positions of the cylinders and difference

streamwise positions of the flow field. The DPIV measurement was conducted at the mid-span of the cylinders. The observation area covered about $210\text{mm}\times 210\text{mm}$ in $L/D = 3.5$ case and correspondingly varied according to L/D . The interrogation window is 32×32 pixels. The wind tunnel setup and LDA and DPIV technique are described in detail by Lam et al. (2002; 2003; 2004; 1995; 1992).

3. Results and discussion

The normalized mean streamwise velocity (U/U_∞) in the near wake of the four cylinders in an in-line square configuration with the spacing ratio (L/D) = 1.5, 2.5, 3.5 and 5.0 at $Re=1.554\times 10^4$ obtained from LDA measurements are shown in Fig. 2. The non-dimensional distance (x/D) from the center of the four cylinders configuration varied with different L/D .

At $L/D = 1.5$ and $x/D = 2.25$ [Fig. 2(a)], it can be clearly seen that the velocity value is positive at $y/D = -0.75$ plane and negative at $y/D = 0.75$ plane. This implies the flow is still in the reverse flow region behind the downstream cylinder ($y/D = 0.75$) and the vortex formation length is different for the two downstream cylinders. It suggests that the flow structure behind the downstream cylinders is distinctly biased to one side and it exhibits a bistable state of a wide and narrow wake. From further $x/D = 4.25$ to $x/D = 11.25$, the normalized mean streamwise velocity distribution indicates that the wake structure from the downstream cylinder 3 and 4 formed an amalgamated structure with the wide wake dominating the narrow wake. This phenomenon of wake characteristics was powerfully supported by the visualization results obtained from DPIV [Fig. 3(b)]. Figure 3 presents the instantaneous vorticity field and velocity field measured by DPIV at the $L/D = 1.5$ and $Re=1.128\times 10^4$, 1.554×10^4 and 1.982×10^4 . It is found that the free shear layers from the upstream cylinders shield the corresponding downstream cylinders completely at all Re . The intermittent change of the wide and narrow wakes behind the downstream cylinders is also observed. However, this bistable nature of the wake flow is completely random. The wake dynamics associated with the amalgamation and the squeezing effect of the vortices behind the downstream cylinders at this configuration are clearly demonstrated. It should be noted that the deflection angle of the wake structure behind the downstream cylinders appears sensitive to the variation in Re and

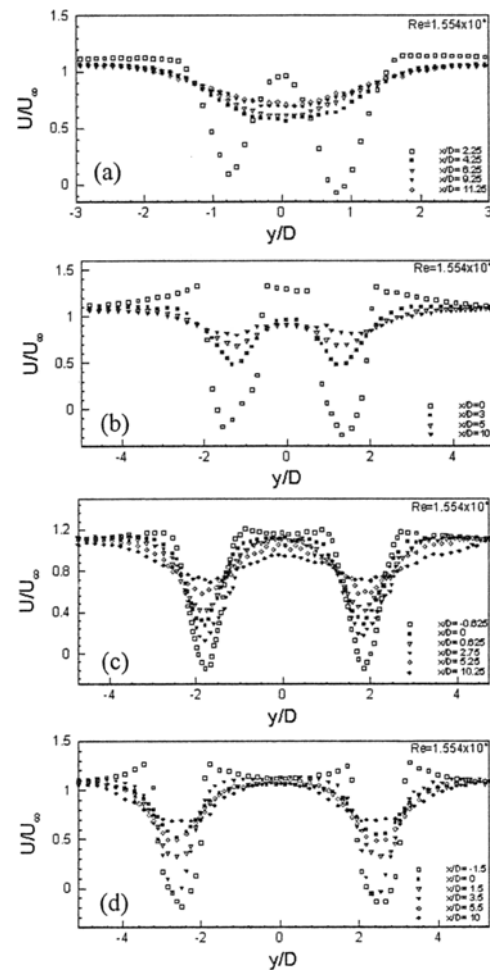


Fig. 2. Comparison of the mean streamwise velocity distributions derived from LDA at different x -positions with $Re=1.554\times 10^4$, (a) $L/D = 1.5$, (b) $L/D = 2.5$, (c) $L/D = 3.5$, (d) $L/D = 5.0$.

decreases with increasing Re . The vortex formation length for the downstream cylinder decreases as Re increases. This wake structure is well consistent with the experimental results by Lam et al. (1992) at $L/D = 1.54$ and $Re = 2100$.

For the $L/D = 2.5$ case [Fig. 2(b)], the normalized mean streamwise velocity distributions behind the two downstream cylinder are found to be almost the same at all x/D . The velocity value is negative at $x/D = 0$. The free shear layers from the upstream cylinder shield or reattach or roll up very near the downstream cylinders depending on Re and the wake structure behind the downstream cylinders is symmetrical. The interference effect of wake regions on one another is not observed at this spacing ratio and at

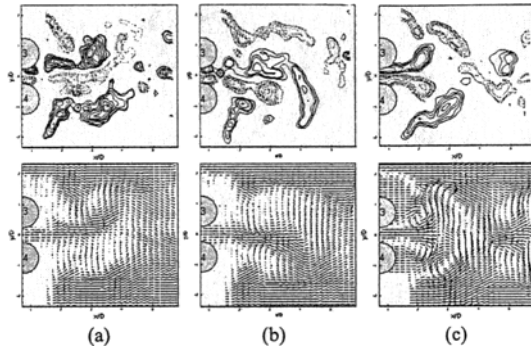


Fig. 3. Instantaneous vorticity field and velocity field derived from DPIV at $L/D = 1.5$, (a) $Re = 1.128 \times 10^4$, (b) $Re = 1.554 \times 10^4$, (c) $Re = 1.982 \times 10^4$, Solid lines are positive vorticity levels and dashed lines are negative.

other higher spacing ratios.

For the critical $L/D = 3.5$ case [Fig. 2(c)], at $x/D = -0.625$ plane, the velocity value is negative (reaching about -0.2 and -0.1 for $Re = 1.554 \times 10^4$ and 1.982×10^4 , respectively) and at $x/D = 0$ and 0.625 planes, the velocity value is positive in $y/D = \pm 1.75$ planes at $Re = 1.554 \times 10^4$ and 1.982×10^4 . But at $Re = 1.1 \times 10^4$, the maximum reverse flow region occurs at $x/D = 0.625$ plane (reaching about -0.4) and at $x/D = 0$ and 0.625 planes, the velocity values are about zero in $y/D = \pm 1.75$ planes. It indicates that the flow behind the upstream cylinders is in the high reverse flow region and the vortex shedding of the upstream cylinders occurs at $Re = 1.554 \times 10^4$ and 1.982×10^4 and the shear layer of the upstream cylinders rolled up very near the downstream cylinder at $Re = 1.128 \times 10^4$. Furthermore, it also suggests that there is a decrease in the vortex formation length behind the upstream cylinders as Re varied from 1.554×10^4 to 1.982×10^4 . The normalized mean streamwise velocity (U/U_∞) along $y/D = -1.75$ plane at $L/D = 3.5$, $Re = 1.982 \times 10^4$, 1.554×10^4 , and 1.128×10^4 are shown in Fig. 4. It indicates again that the flow pattern transformation between the four cylinders depends on Re at $L/D = 3.5$. It can also be seen that the vortex formation length (same as Lam (2004)'s definition) for the downstream cylinders decreases as Re decreases. On the other hand, there is a decrease of about 16% in the vortex formation length for the upstream cylinders as Re increases from 1.554×10^4 to 1.982×10^4 .

Figure 5 presents that instantaneous vorticity field derived from DPIV at the $L/D = 3.5$ and $Re = 1.128 \times 10^4$, 1.554×10^4 and 1.982×10^4 . It can be clearly observed that depending on Re , the shear layers from upstream

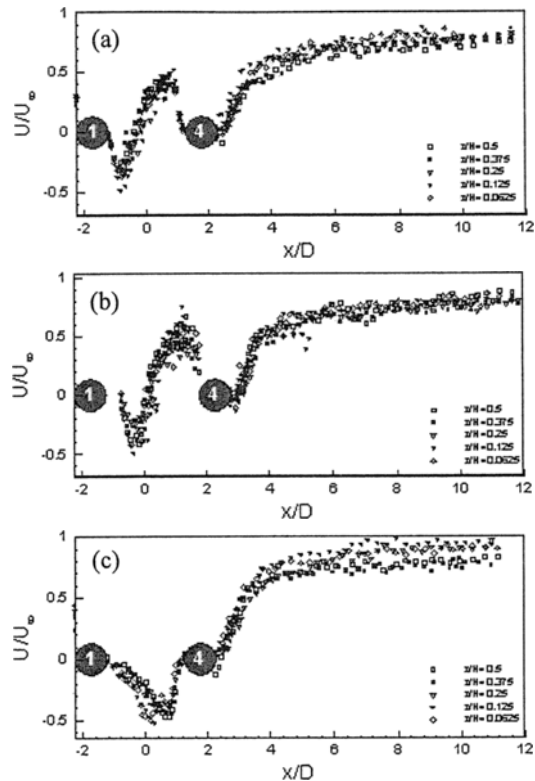


Fig. 4. Mean streamwise velocity at different spanwise positions derived from LDA along $y/D = -1.75$ at $L/D = 3.5$, (a) $Re = 1.982 \times 10^4$, (b) $Re = 1.554 \times 10^4$, (c) $Re = 1.128 \times 10^4$.

cylinders rolled up very near the downstream cylinder at $Re = 1.128 \times 10^4$ [Fig. 5(a)] and roll up into mature vortices and impinge on the downstream cylinder surface at $Re = 1.554 \times 10^4$ and $Re = 1.982 \times 10^4$ [Fig. 5(b), (c)]. Furthermore, at $Re = 1.554 \times 10^4$ and 1.982×10^4 , the flow behavior such as in-phase vortex shedding, anti-phase vortex shedding and synchronized vortex shedding from the upstream cylinders are also observed using the DPIV technique.

For the $L/D = 5.0$ case [Fig. 2(d)], at the $x/D = -1.5$, 0 and 1.5 plane, the normalized mean streamwise velocity distributions exhibits a similar phenomenon as $L/D = 3.5$ at $Re = 1.554 \times 10^4$. It proposes that the flow behind the upstream cylinders is in the high reverse flow region and occurs a similar flow pattern where the shear layer from upstream cylinders roll up into mature vortices and impinge on the downstream cylinder surface at all Re . It is interesting that the mean streamwise velocity distributions at $x/D = 3.5$ (the distance from the centre of the downstream cylinders is about $1D$) is very similar to that at $x/D = -1.5$ (the distance from the centre of the

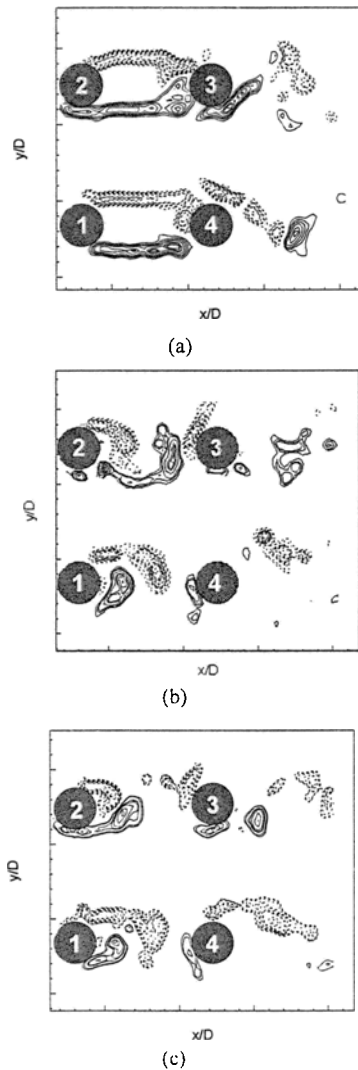


Fig. 5. Instantaneous vorticity field derived from DPIV at $L/D=3.5$, (a) $Re=1.128 \times 10^4$, (b) $Re=1.554 \times 10^4$, (c) $Re=1.982 \times 10^4$, Solid lines are positive vorticity levels and dashed lines are negative.

upstream cylinders is about $1D$) and are close to that of a single cylinder. It is consistent with the flow visualization experimental result obtained by Lam et al. (1992).

The 3-D numerical simulations of the cross-flow around four cylinders in an in-line square configuration with the spacing ratio 1.5 and 3.5 at $Re=1.50 \times 10^4$ have also been carried out using large eddy simulation (LES). The numerical models agree with the configuration of the experiment study. A periodic boundary condition is employed at the both ends of the cylinders and 'no-slip' condition is used at the

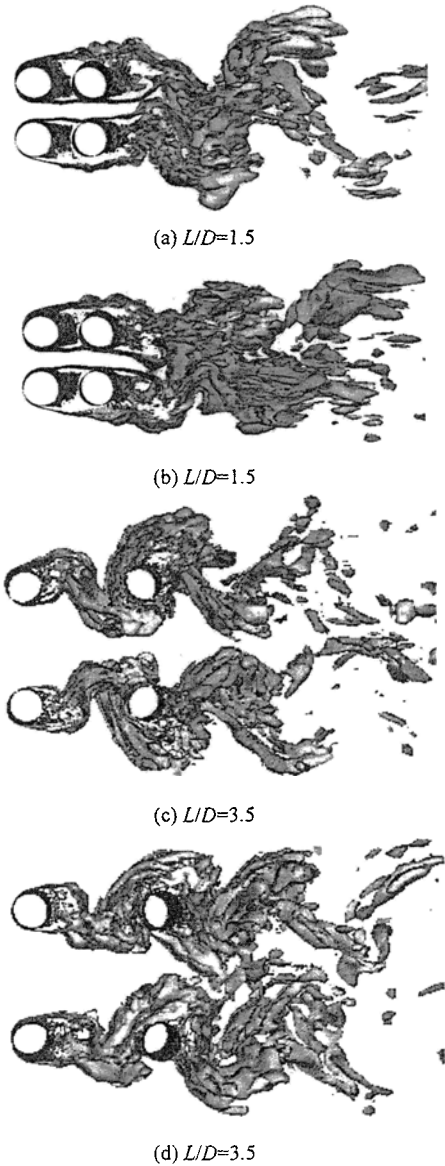


Fig. 6. The velocity fields over the iso-vorticity surfaces for the four cylinders at $L/D=1.5$ and 3.5 , $Re=1.50 \times 10^4$, (a) deflected upwards, (b) deflected downwards, (c) anti-phase vortex shedding, (d) in-phase vortex shedding.

cylinder surfaces. Figure 6 presents that the velocity fields over the iso-vorticity surfaces for the four cylinders with $L/D=1.5$ and 3.5 at $Re=1.50 \times 10^4$. The computations captured accurately the 3-D vortices structures behind the downstream cylinders of the different biased orientations at the $L/D=1.5$ case [Fig. 6(a), (b)] as well as 3-D vortices structures between the four cylinders of the anti-phase vortex shedding [Fig. 6(c)] and in-phase vortex shedding [Fig. 6(d)] at

the $L/D = 3.5$ case. It was found to be in good qualitative agreement with the observed flow visualization results derived from DPIV at the same Re . The 3-D numerical simulation using LES well reproduces the experimental features of the complex cross-flow around the four cylinders hence reveals full field quantitative information which may not be very easy to obtain by experimental investigation.

4. Conclusions

The cross-flow around the four cylinders in an in-line square configuration with varying spacing ratio in range of 1.5–5.0 at a sub-critical Reynolds number are measured using LDA and DPIV and are simulated three-dimensionally using LES. The major results are summarized as follows:

Depend on the spacing ratio and Re , several distinct flow patterns were observed. The remarkable resemblance of the bistable nature of the flow was observed especially at $L/D = 1.5$ in experimental study and numerical simulation and the deflection angle of the wake structure decreases with increasing Re . Symmetrical flow occurred for higher spacing ratios ($L/D \geq 2.5$). And distinct vortex shedding of the upstream cylinders was suppressed for $L/D < 3.5$ at a sub-critical Reynolds number.

The vortex formation length for the upstream and downstream cylinders is affected by the spacing ratio and Re . For the critical $L/D = 3.5$ case, there is a decrease of about 16% in the vortex formation length for the upstream cylinders as Re increases from 1.554×10^4 to 1.982×10^4 . The wider wake structure behind the upstream and downstream cylinders occurred at the lower Re .

3-D numerical simulation using LES well reproduces the experimental features of the flow field and reveals full field quantitative information which may not be very easy to obtain by experimental investigation.

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